Potential and Challenges for a Certified Application of Model Reference Adaptive Control to Aerial Vehicles

Mattia Gramuglia[1], Giri M. Kumar[1], and Andrea L'Afflitto[2]

***Abstract*— The model reference adaptive control (MRAC) framework has proven in an academic context its ability to support complex applications and assure successful results in off-nominal flight conditions. However, this technology is far from being systematically implemented in industrial applications. This paper reflects on some of the challenges hindering this systematic dissemination, and proposes some solutions to be shared by the academic, defense, and industrial communities.**

***Index Terms*— Adaptive control, Multi-rotor UAVs, Flight stack.**

I. Introduction

Model reference adaptive control (MRAC) is a cornerstone of adaptive control theory [1]. In its most basic form, an MRAC controller steers the trajectory of a plant affected by parametric, matched, and unmatched uncertainties toward the trajectory of a reference model. However, the MRAC design philosophy is not to merely steer the plant trajectory toward a reference signal, *i.e.*, the state of the reference model, but to impose that the controlled plant dynamics are those of a user-defined, linear, bounded-input bounded-state dynamical model, *i.e.*, the reference model, despite uncertainties.

Over the years, numerous variations of classical MRAC have been proposed. Some of these variations involve the robustifications of MRAC; among these variations we recall the use of the dead-zone modification [2], the σ-modification [3], the e-modification [4], the convex projection operator [5], and the use of barrier Lyapunov function on the adaptive gains [6], [7]. Other variations of MRAC enable output signal tracking [1, Ch. 10], impose user-defined constraints on the trajectory tracking error [8]–[10], and allow the control of switched [11] and hybrid [12]–[14] plant models. A latest trend in MRAC theory is embedding machine learning [15], [16] and data-driven methods [17], [18].

MRAC has been applied to a myriad of problems of practical interest and tested in numerical simulations as well as field tests. However, these tests have been primarily conducted in academic settings. MRAC and several of its classical variations have been implemented also on autonomous systems employed in defense applications, such as uncrewed aerial vehicles (UAVs). However, despite the plethora of theoretical results, examples, and successful implementations, this technology still seems far from being transitioned systematically to the defense technology and ever employed in civilian applications.

[1]M. Gramuglia and G. M. Kumar are with Virginia Tech, Department of Mechanical Engineering, Blacksburg, VA 24061, USA

[2]Andrea L'Afflitto is with the Department of Industrial and Systems Engineering of Virginia Tech, Blacksburg, VA, USA (email: a.lafflitto@vt.edu)

This paper is structured as follows. In Section II, we provide a brief summary of the MRAC technology. Successively, in Section III, we recall how MRAC can be applied to design control systems for multi-rotor UAVs such as quadcopters or similar configurations. Section IV presents the results of some flight tests involving an X8-copter. These tests clearly show the superiority of MRAC to classical solutions based on feedback linearization and proportional-integral-derivative (PID) controllers while transporting a heavy, unsteady payload. Finally, Section V draws conclusions, reflects on why MRAC systems, and other advanced control systems, are not being employed systematically in defense and industrial settings, and outlines possible solutions.

II. Elements of Robust MRAC

The MRAC design problem can be summarized as follows. Consider the plant model

$$\dot{x}(t) = Ax(t) + B\Lambda\left[u(t) + \Theta^{\mathrm{T}}\Phi(t, x(t))\right] + \xi(t), \quad x(t_0) = x_0, \qquad t \geq t_0, \quad (1)$$

where $x : [t_0, \infty) \to \mathbb{R}^n$ denotes the *plant state*, $u : [t_0, \infty) \to \mathbb{R}^m$ denotes the *control input*, $\Phi : [t_0, \infty) \times \mathbb{R}^n \to \mathbb{R}^N$ denotes the *regressor vector*, $A \in \mathbb{R}^{n\times n}$ is unknown, $B \in \mathbb{R}^{n\times m}$ is known and such that the pair $(A, B\Lambda)$ is controllable, $\Lambda \in \mathbb{R}^{m\times m}$ is diagonal, positive-definite, and unknown, and $\Theta \in \mathbb{R}^{N\times m}$ is unknown and such that $\Theta^{\mathrm{T}}\Phi(t, x)$ captures matched uncertainties for all $(t, x) \in [t_0, \infty) \times \mathbb{R}^n$, and $\xi : [t_0, \infty) \to \mathbb{R}^n$ is continuous and uniformly bounded. The matrices A, Λ, and Θ capture parametric uncertainties; the term $\Theta^{\mathrm{T}}\Phi(t, x)$, $(t, x) \in [t_0, \infty) \times \mathbb{R}^n$, captures matched uncertainties in the plant's nonlinearities, and $\xi(t)$ captures unmatched uncertainties, such as, for instance, external disturbances.

MRAC systems steer the plant trajectory $x(\cdot)$ to follow the trajectory of the *reference model*

$$\dot{x}_{\mathrm{ref}}(t) = A_{\mathrm{ref}}x_{\mathrm{ref}}(t) + B_{\mathrm{ref}}r(t), \quad x_{\mathrm{ref}}(t_0) = x_{\mathrm{ref},0}, \qquad t \geq t_0, \quad (2)$$

where $x_{\mathrm{ref}} : [t_0, \infty) \to \mathbb{R}^n$ denotes the *reference trajectory*, $r : [t_0, \infty) \to \mathbb{R}^m$ is user-defined, continuous, uniformly bounded, and denotes the *reference command input*, $A_{\mathrm{ref}} \in \mathbb{R}^{n\times n}$ is user-defined Hurwitz, $B_{\mathrm{ref}} \in \mathbb{R}^{n\times m}$ is user-defined, and the matching conditions

$$A_{\mathrm{ref}} = A + B\Lambda K_x^{\mathrm{T}}, \quad B_{\mathrm{ref}} = B\Lambda K_r^{\mathrm{T}} \quad (3)$$

are verified by some $K_x \in \mathbb{R}^{n\times n}$ and $K_r \in \mathbb{R}^{n\times m}$. MRAC laws are in the form

$$\phi(\pi(t), \hat{K}(t)) = \hat{K}^{\mathrm{T}}(t)\pi(t), \qquad t \geq t_0, \quad (4)$$

where $\pi(t) \triangleq \left[x^{\mathrm{T}}(t), r^{\mathrm{T}}(t), -\Phi^{\mathrm{T}}(t,x(t))\right]^{\mathrm{T}}$ and the *adaptive gain* $\hat{K} : [t_0,\infty) \to \mathbb{R}^{(n+m+N)\times m}$ verifies one of the following alternative adaptive laws

$$\dot{\hat{K}}(t) = \mathrm{dead}(e(t))\left(-\Gamma\pi(t)e^{\mathrm{T}}(t)PB\right) \quad \hat{K}(t_0) = \hat{K}_0, \tag{5a}$$

$$\dot{\hat{K}}(t) = -\Gamma\pi(t)e^{\mathrm{T}}(t)PB - \sigma\Gamma\hat{K}(t), \quad \hat{K}(t_0) = \hat{K}_0, \tag{5b}$$

$$\dot{\hat{K}}(t) = -\Gamma\pi(t)e^{\mathrm{T}}(t)PB - \sigma\Gamma\|e^{\mathrm{T}}(t)PB\|\hat{K}(t), \quad \hat{K}(t_0) = \hat{K}_0, \tag{5c}$$

where $\Gamma \in \mathbb{R}^{(n+m+N)\times(n+m+N)}$ is symmetric and positive definite, $P \in \mathbb{R}^{n\times n}$ denotes the symmetric positive-definite solution of

$$-Q = A_{\mathrm{ref}}^{\mathrm{T}}P + PA_{\mathrm{ref}}, \tag{6}$$

$Q \in \mathbb{R}^{n\times n}$ is user-defined, symmetric, and positive-definite, $\sigma > 0$, $\mathrm{dead} : \mathbb{R}^{(n+m+N)\times m} \times \mathbb{R}^n \to \mathbb{R}^{(n+m+N)\times m}$ is such that

$$\mathrm{dead}(e) \triangleq \begin{cases} 1 & \text{if } \|e\| > d, \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

and $d > 0$ is user-defined. The function (7) is discontinuous. Thus, (7) is usually replaced with a continuous or continuously differentiable approximating function. It is important to remark that the adaptive laws in (5) are alternative to one another. However, (5a) and (5b) or (5a) and (5c) can be combined to activate the adaptive law only past a user-defined threshold.

It follows from (1)–(4) that the closed-loop trajectory tracking error dynamics is given by

$$\dot{e}(t) = A_{\mathrm{ref}}e(t) + B\Lambda\Delta K^{\mathrm{T}}(t)\pi(t) + \xi(t), \quad e(t_0) = x_0 - x_{\mathrm{ref},0}, \quad t \geq t_0, \tag{8}$$

where $\Delta K(t) \triangleq \hat{K}(t) - K$. Thus, the effectiveness of MRAC systems can be summarized as follows.

Theorem 1: Consider the dynamical model given by (8) and one of the adaptive laws in (5). The closed-loop trajectory tracking error dynamics (8) are uniformly ultimately bounded and the adaptive gain $\hat{K}(\cdot)$ is uniformly bounded.

Corollary 2.1: Consider the dynamical model given by (8) with $\xi(t) \equiv 0$, $t \geq t_0$ and one of the adaptive laws in (5) with $\sigma = 0$ and $d = 0$. The closed-loop trajectory tracking error dynamics (8) are asymptotically convergent to zero and the adaptive gain $\hat{K}(\cdot)$ is uniformly bounded.

III. MRAC Systems for X8-Copters

X8-copters are multi-rotor UAVs equipped with four push propellers and four pull propellers placed pair-wise at the extremities of a quadcopter frame. The X8-copter configuration is advantageous for a higher thrust-to-volume ratio compared to other multi-rotor UAVs.

To design the control architecture for an X8-copter UAV, such as the one shown in Figure 1, we recall that its translational dynamics are captured by

$$\dot{r}^{\mathbb{I}}(t) = v^{\mathbb{I}}(t), \qquad r^{\mathbb{I}}(t_0) = r_0^{\mathbb{I}}, \quad t \geq t_0, \tag{9a}$$

Fig. 1: X8-copter employed for the proposed flight tests

$$m\dot{v}^{\mathbb{I}}(t) = -R(\alpha(t))\left[u_1(t)e_3 + \frac{1}{2}\rho_{\mathrm{air}}Sc_{\mathrm{D}}\|v(t)\|v(t)\right] + mge_3, \quad v^{\mathbb{I}}(t_0) = v_0^{\mathbb{I}}, \tag{9b}$$

where $r^{\mathbb{I}} : [t_0,\infty) \to \mathbb{R}^3$ denotes the position of the vehicle's center of mass in the inertial reference frame $\mathbb{I}$ with axes X, Y, and Z, $v^{\mathbb{I}} : [t_0,\infty) \to \mathbb{R}^3$ denotes the velocity of the vehicle center of mass in $\mathbb{I}$, $v(\cdot)$ denotes the UAV velocity expressed in the body reference frame $\mathbb{J}$ with axes x, y, and z, $m > 0$ denotes the mass of the UAV *without* payload, $\alpha(\cdot) \triangleq [\phi(\cdot), \theta(\cdot), \psi(\cdot)]^{\mathrm{T}}$,

$$R(\alpha) \triangleq \begin{bmatrix} \mathrm{c}\psi & -\mathrm{s}\psi & 0 \\ \mathrm{s}\psi & \mathrm{c}\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{c}\theta & 0 & \mathrm{s}\theta \\ 0 & 1 & 0 \\ -\mathrm{s}\theta & 0 & \mathrm{c}\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathrm{c}\phi & -\mathrm{s}\phi \\ 0 & \mathrm{s}\phi & \mathrm{c}\phi \end{bmatrix},$$

captures a rotation matrix, $\phi : [t_0,\infty) \to [-\pi,\pi)$ $\theta : [t_0,\infty) \to (-\frac{\pi}{2},\frac{\pi}{2})$, $\psi : [t_0,\infty) \to [-\pi,\pi)$ denote the roll, pitch, and yaw angles of a 3-2-1 rotation sequence, respectively, $u_1 : [t_0,\infty) \to \mathbb{R}$ denotes the total thrust generated by the propellers, $g > 0$ denotes the gravitational acceleration, and $e_3 \triangleq [0,0,1]^{\mathrm{T}}$. The air density $\rho_{\mathrm{air}} > 0$, the UAV's cross-section area $S > 0$, and the diagonal, positive-definite drag coefficient matrix $c_{\mathrm{D}} \in \mathbb{R}^{3\times3}$ are unknown. The rotational dynamics are given by

$$\dot{\alpha}(t) = \Gamma_{\mathrm{J}}(\phi(t),\theta(t))\omega(t), \quad \alpha(t_0) = \alpha_0, \quad t \geq t_0, \tag{10a}$$

$$I\dot{\omega}(t) = \tau(t) - \omega^{\times}(t)I\omega(t), \quad \omega(t_0) = \omega_0, \tag{10b}$$

where $\omega : [t_0,\infty) \to \mathbb{R}^3$ denotes the *angular velocity* relative to an inertial reference frame,

$$\Gamma_{\mathrm{J}}(\phi,\theta) \triangleq \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix}, \tag{11}$$

$I = \mathrm{diag}\left([I_x, I_y, I_z]\right)$ denotes the diagonal, positive-definite matrix of inertia with respect to the center of mass, and $\tau : [t_0,\infty) \to \mathbb{R}^3$ denotes the torque applied to the body. In this paper, if a vector $z \in \mathbb{R}^3$ is expressed in the reference frame $\mathbb{I}$, then it is denoted as $z^{\mathbb{I}}$. If $z \in \mathbb{R}^3$ is expressed in the reference frame $\mathbb{J}$, then it is denoted as z.

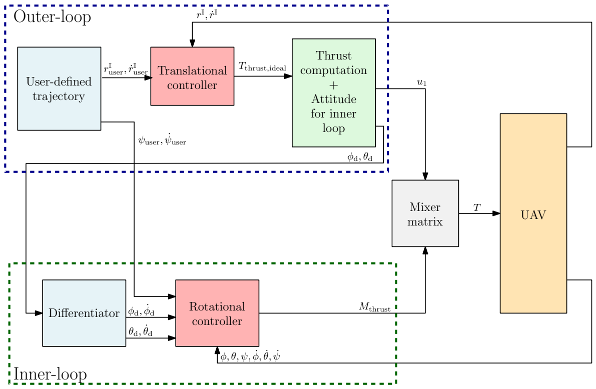

Fig. 2: Block diagram of a control system for X8-copter

A. *Outer loop design*

To resolve the under-actuation of this UAV and determine the required total thrust, an outer loop is employed; see Figure 2. As detailed in [13], given the twice continuously differentiable user-defined reference trajectory $r^{\mathbb{I}}_{\rm user} : [t_0, \infty) \to \mathbb{R}^3$ and a twice continuously differentiable desired yaw angle $\psi_{\rm user} : [t_0, \infty) \to [-\pi, \pi)$, to design the outer loop, we introduce the virtual translational control input $\mu^{\mathbb{I}} : [t_0, \infty) \to \mathbb{R}^3$ which can be designed applying a PID controller with a dynamic inversion baseline or an MRAC system so that the solution of

$$\dot{r}^{\mathbb{I}}(t) = v^{\mathbb{I}}(t), \qquad r^{\mathbb{I}}(t_0) = r^{\mathbb{I}}_0, \quad t \geq t_0, \tag{12a}$$

$$m\dot{v}^{\mathbb{I}}(t) = \mu^{\mathbb{I}}(t) - \frac{1}{2}\rho_{\rm air} S R(\alpha(t)) c_{\rm D} \|v(t)\| v(t) + mge_3, \quad v^{\mathbb{I}}(t_0) = v^{\mathbb{I}}_0, \tag{12b}$$

follows the user-defined trajectory $\left[r^{\rm T}_{\rm user}(t), \dot{r}^{\rm T}_{\rm user}(t)\right]^{\rm T}$. A realization of the virtual control input is given by

$$\mu(t) = -u_1(t) R(\alpha_{\rm d}(t)) e_3, \quad t \geq t_0, \tag{13}$$

where $\alpha_{\rm d} : [t_0, \infty) \to [-\pi, \pi) \times (-\frac{\pi}{2}, \frac{\pi}{2}) \times \{0\}$. The norm of $\mu(\cdot)$ determines the total thrust, and the orientation of $\mu(\cdot)$ determines the desired roll angle $\phi_{\rm d} : [t_0, \infty) \to [-\pi, \pi)$ and the desired pitch angle $\theta_{\rm d} : [t_0, \infty) \to (-\frac{\pi}{2}, \frac{\pi}{2})$ so that $\alpha_{\rm d}(t) \triangleq [\phi_{\rm d}(t), \theta_{\rm d}(t), 0]^{\rm T}$.

It follows from (13) that, for all $t \geq t_0$,

$$u_1(t) = \sqrt{\mu_x^2(t) + \mu_y^2(t) + \mu_z^2(t)}, \tag{14a}$$

$$\theta_{\rm d}(t) = {\rm atan2}(-\mu_x(t), -\mu_z(t)), \tag{14b}$$

$$\phi_{\rm d}(t) = {\rm atan2}\left(\frac{\mu_y(t)}{u_1(t)}, \sqrt{1 - \frac{\mu_y^2(t)}{u_1^2(t)}}\right), \tag{14c}$$

where ${\rm atan2} : (\mathbb{R} \times \mathbb{R}) \setminus \{0\} \to [-\pi, \pi)$ denotes the *signed arc-tangent function*.

B. *Inner loop design*

The inner loop computes the moment of the thrust force $\tau(\cdot)$ needed to realize the desired attitude $[\phi_{\rm d}(t), \theta_{\rm d}(t), \psi_{\rm user}(t)]^{\rm T}$, $t \geq t_0$; see Figure 2. Since the rotational dynamics of a multi-rotor UAV are fully actuated, we can directly control the roll, pitch, and yaw angles with the three components of the torque $\tau(t) = [\tau_x(t), \tau_y(t), \tau_z(t)]^{\rm T}$, $t \geq t_0$. In this work, we let

$$\tau(t) = \omega^{\times}(t) \bar{I} \omega(t) + \bar{I} \tilde{\tau}(t), \qquad t \geq t_0. \tag{15}$$

where $\bar{I} \in \mathbb{R}^{3\times3}$ denotes the estimated inertia matrix of the UAV computed with respect to the center of mass and $\tilde{\tau}(t) = [\tilde{\tau}_x(t), \tilde{\tau}_y(t), \tilde{\tau}_z(t)]^{\rm T}$. The first term on the right-hand side of (15) provides a baseline controller designed according to the dynamic inversion technique, which attempts to cancel the nonlinear term in (10b). The second term on the right-hand side of (15) can be designed employing a PID controller or an MRAC system so that

$$\lim_{t\to\infty} \|\alpha(t) - [\phi_{\rm d}(t), \theta_{\rm d}(t), \psi_{\rm user}(t)]^{\rm T}\| = 0$$

despite the fact that $\bar{I} \neq I$; for details, see [13].

C. *Thrust realization*

Having deduced the total thrust $u_1(\cdot)$ and the moment of the thrust $\tau(\cdot)$, we compute the thrust required by each motor as

$$T(t) = M^{+} \left[u_1(t), \tau(t)\right]^{\rm T}, \qquad t \geq t_0, \tag{16}$$

where

$$M \triangleq \begin{bmatrix} 1 & l_y & -l_x & -c_T \\ 1 & l_y & l_x & c_T \\ 1 & -l_y & l_x & -c_T \\ 1 & -l_y & -l_x & c_T \\ 1 & l_y & -l_x & c_T \\ 1 & l_y & l_x & -c_T \\ 1 & -l_y & l_x & c_T \\ 1 & -l_y & -l_x & -c_T \end{bmatrix}^{\rm T}, \tag{17}$$

$l_x > 0$ denotes the distance between each motor and the pitch axis, $l_y > 0$ denotes the distance between each motor and the roll axis, $c_T > 0$ denotes the propellers' drag coefficient, M^{+} denotes the *Moore-Penrose pseudoinverse* of M, and $T(t) \triangleq [T_1(t), \ldots, T_8(t)]^{\rm T}$. Figure 2 shows the relationship between outer loop, inner loop, and thrust realization for an X8-copter.

IV. Flight Test Results

In this section, we present the results of two flight tests performed on the X8-copter shown in Figure 1. In these tests, the UAV is tasked with transporting a steel ball that is free to move in its casing. The mass of the ball is 0.13kg, that is, 6.31% of the UAV's mass, and the mass of the casing is 0.52kg. One of these flight tests is performed employing a feedback-linearizing and a PID control law in both the outer loop and the inner loop. The other flight test is performed augmenting the previous control system with a robust MRAC system obtained by combining (5a) with (5c) in the outer loop. The control systems employed in both tests have been tuned without the ball and without its casing.

The proposed flight tests were performed employing the freeware, open-source, PX4-compatible, offboard flight stack for multi-rotor UAVs produced by the authors [19]. This flight stack implements the control system architecture shown in Figure 2 and is executed on an Odroid M1S

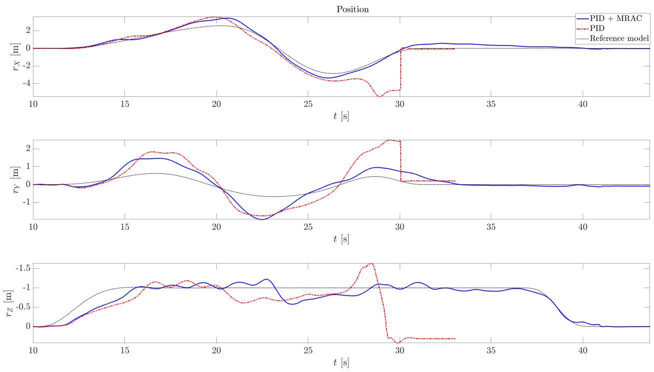

Fig. 3: Components of the UAV's position in a North-East-Down inertial reference frame. Employing a feedback linearizing and a PID control law, the vehicle is unable to complete its mission. Augmenting this control law with a robust MRAC law in the outer loop, the vehicle closely follows the user-defined trajectory and completes its mission

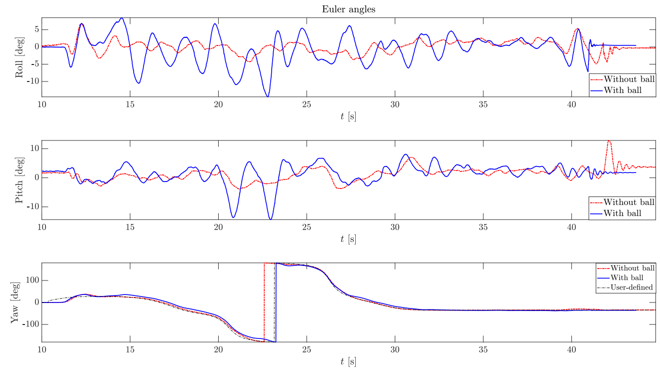

Fig. 4: Roll, pitch, and yaw angles of the UAV obtained employing the robust MRAC law in the outer loop while transporting the steel ball, and Euler angles obtained employing the robust MRAC law in the outer loop without transporting the steel ball. The unsteady payload induces larger and faster variations in the pitch and roll angles

single-board computer interfaced with a Pixhawk 6c flight controller. Information on the UAV's state by blending information from a VICON motion capture system and the flight controller.

Figure 3 shows the UAV position in both flight tests and the user-defined reference trajectory. Employing a feedback linearizing and a PID control law, the vehicle is unable to complete its mission. Augmenting this control law with a robust MRAC law in the outer loop, the vehicle follows the user-defined trajectory and completes its mission.

Figure 4 shows the Euler angles of the UAV employing the robust MRAC law in the outer loop both in nominal and in off-nominal conditions, that is, without and with the ball and its casing. The presence of an unsteady payload challenges the control system with larger and faster variations in the pitch and roll angles.

Videos and additional plots of the proposed flight tests are shown in [20]. Reference [20] also shows the results of a flight test obtained by applying the same feedback-linearizing and PID control laws in nominal conditions. In this mission, the user-defined trajectory is considerably more aggressive than the one shown in Figure 3. This flight test, which is not discussed in this section for brevity, shows how the feedback-linearizing and PID control laws have been tuned to obtain very close trajectory following performance in nominal conditions.

V. Concluding Remarks

The flight tests presented in Section IV show how the MRAC technology allows performing with success missions in off-nominal conditions. A more relevant aspect of the MRAC technology is that it is not an alternative to existing control systems, such as those currently employed in commercial-off-the-shelf autopilots. MRAC systems can be employed to augment existing control systems, which, therefore, do not need to be re-tuned. Indeed, as shown in [13], any pre-existing controller can be employed as a baseline controller for MRAC, and tuning (7) allows MRAC systems to intervene only when needed.

Thus, why is the MRAC technology not yet widely used? The answers to this question are multiple. Some of the most challenging ones are the following. Firstly, insurance companies do not have enough historic data to statistically assess the reliability of these systems. Secondly, there are no verification and validation (V&V) tools and methodologies to analyze adaptive controller stability and convergence. Finally, there are no metrics to assess the performance of MRAC systems in off-nominal flight conditions.

While these problems are sufficiently relevant to significantly slow the systematic use of Group 1 autonomous UAVs equipped with adaptive control systems in the National Air Space (NAS) for tasks such as payload delivery, the same problems drastically halt the certification process for higher groups of autonomous UAVs, urban air mobility (UAM) systems, and crewed air vehicles. To certify an adaptive control system on a commercial aircraft in the U.S., the flight software must substantially comply with RTCA DO-178C [21]. However, without historic data, there is no comparison term to prove air worthiness of adaptive control systems. While commenting on RTCA DO-178B [22], the predecessor of DO-178C, the author of [23] remarked on how these guidelines are in terms of high-level requirements from the standpoint of control researchers. Additionally, there is overly little awareness of nonlinear systems theory in general and adaptive control systems theory in particular among designated engineering representative (DER) and Federal Aviation Administration (FAA) officials for them to certify adaptive controllers with absolute confidence.

The author in [23] proposed several community-led efforts to eventually certify adaptive control systems systematically. Such approaches include the use of high fidelity simulations, alternatives to Lyapunov-based tools, metrics that generalize phase and gain margins to nonlinear, time-varying systems, and probabilistic uncertainty models.

The introduction of a theoretical framework alternative to the current one, which is radically based on Lyapunov-like arguments, seems rather improbable in the short term. Long-established, mathematically sound, alternative approaches to characterize "well-behaved" dynamical models, such as dissipativity theory to name one, are far from being as popular as Lyapunov theory. It seems therefore unlikely that the use of a framework alternative to Lyapunov theory would be a viable approach to certify adaptive control systems.

The idea of commonly accepted high-fidelity simulation tools, the use of standard hardware and software architectures, and the use of extensive databases that would serve as a golden standard seem more feasible approaches. Decades of experience in finite element analysis has lead to very reliable tools. In [10], [13], [14], the authors employed Chrono [24] for the first time in the context of UAV control. This software, which is popular in the domain of off-road ground vehicle analysis, has been chosen for being open-source, high fidelity and high computational speed, and not relying on costly hardware setups. A tool as mature as Chrono may become a simulation tool also in the UAV and UAM communities.

Although there are relatively few variations of the hardware architectures of UAV control systems, there is no consensus on the software architecture to control UAVs. PX4 and ArduPilot provide relatively standard architectures for the flight controllers. However, it is safe to affirm that almost each research lab has its own, undisclosed, flight stack architecture to be executed on a companion computer, and more sophisticated control systems are usually executed off-board. Recently, the authors disclosed [19], whose ambition is to promote a community-led effort toward the production of a widely accepted, PX4-compatible, software architecture for companion computers. To the authors' knowledge there is no similar initiative in the UAV community.

To the authors' knowledge, there are no databases of flight results. For such a tool to be useful and enable the certification of adaptive control systems, standard hardware and software architectures, standard missions, and very rich sets of data points would be needed; these conditions appear to require substantial effort. On the other hand, the UAV community is particularly vast, in constant growth, and sufficiently vibrant to envision such a tool. Considering that said arguments transcends adaptive control theory and substantially apply to any nonlinear control technique aiming to be certified for aeronautical applications, government-supported efforts would serve as an outstanding catalyzer.

ACKNOWLEDGMENT

This research was in part performed with the support of NSF through the Grant no. 2137159 and the US Army Research Lab under Grant no. W911QX2320001.

REFERENCES

[1] E. Lavretsky and K. Wise, *Robust and Adaptive Control: With Aerospace Applications*. London, UK: Springer, 2012.

[2] J.-J. E. Slotine and J. A. Coetsee, "Adaptive sliding controller synthesis for non-linear systems," *International Journal of Control*, vol. 43, no. 6, pp. 1631–1651, 1986.

[3] P. Ioannou and P. Kokotovic, *Adaptive Systems with Reduced Models*. New York, NY: Springer, 1983.

[4] K. Narendra and A. Annaswamy, "A new adaptive law for robust adaptation without persistent excitation," *IEEE Transactions on Automatic Control*, vol. 32, no. 2, pp. 134–145, 1987.

[5] P. Ioannou and B. Fidan, *Adaptive Control Tutorial*. Philadelphia, PA: Society for Industrial and Applied Mathematics, 2006.

[6] A. L'Afflitto and T. A. Blackford, "Constrained dynamical systems, robust model reference adaptive control, and unreliable reference signals," *International Journal of Control*, vol. 93, no. 5, pp. 1039–1052, 2020.

[7] A. L'Afflitto, "Barrier Lyapunov functions and constrained model reference adaptive control," *IEEE Control Systems Letters*, vol. 2, no. 3, pp. 441–446, 2018.

[8] T. Yucelen, G. D. L. Torre, and E. N. Johnson, "Improving transient performance of adaptive control architectures using frequency-limited system error dynamics," *International Journal of Control*, vol. 87, no. 11, pp. 2383–2397, 2014.

[9] R. B. Anderson, J. A. Marshall, and A. L'Afflitto, "Novel model reference adaptive control laws for improved transient dynamics and guaranteed saturation constraints," *Journal of the Franklin Institute*, vol. 358, no. 12, pp. 6281–6308, 2021.

[10] M. Gramuglia, G. M. Kumar, and A. L'Afflitto, "Two-layer adaptive funnel MRAC with applications to the control of multi-rotor UAVs," in *International Workshop on Robot Motion and Control*, pp. 31–36, 2024.

[11] R. B. Anderson, J. A. Marshall, A. L'Afflitto, and J. M. Dotterweich, "Model reference adaptive control of switched dynamical systems with applications to aerial robotics," *Journal of Intelligent & Robotic Systems*, vol. 100, pp. 1265–1281, Dec 2020.

[12] A. L'Afflitto, "Model reference adaptive control for nonlinear time-varying hybrid dynamical systems," *International Journal of Adaptive Control and Signal Processing*, 2023. In press.

[13] M. Gramuglia, G. M. Kumar, and A. L'Afflitto, "A hybrid model reference adaptive control system for multi-rotor unmanned aerial vehicles," in *SciTech*, (Orlando, FL), pp. 1–21, AIAA, 2024.

[14] M. Gramuglia, G. M. Kumar, and A. L'Afflitto, "Adaptive control for hybrid dynamical systems with user-defined rate of convergence," *Journal of the Franklin Institute*, vol. 361, no. 9, p. 106854, 2024.

[15] G. Joshi and G. Chowdhary, "Deep model reference adaptive control," in *Conference on Decision and Control*, pp. 4601–4608, 2019.

[16] G. Joshi, G. Chowdhary, and B. van Bloemen Waanders, "Stochastic deep model reference adaptive control," in *IEEE Conference on Decision and Control*, pp. 1075–1082, 2021.

[17] H. Wang, B. Scurlock, N. Powell, A. L'Afflitto, and A. J. Kurdila, "MRAC with adaptive uncertainty bounds via operator-valued reproducing kernels," *IEEE Control Systems Letters*, vol. 7, pp. 3771–3776, 2023.

[18] J. A. Kurdila, A. L'Afflitto, and J. A. Burns, *Robust and Adaptive Control of Nonlinear Systems over Native Spaces*. London, UK: Springer, 2024. In press.

[19] M. Gramuglia, G. M. Kumar, and A. L'Afflitto, "The Advanced Control Systems Lab Fligth Stack." `https://github.com/andrealaffly/ACSL-flightstack`.

[20] M. Gramuglia, G. M. Kumar, and A. L'Afflitto, "Flight test results employing the Advanced Control Systems Lab flight stack." `https://www.youtube.com/watch?v=Ykjjg21iAm0`, 2024. Last accessed 07/30/2024.

[21] "Software considerations in airborne systems and equipment certification," 2012. Document No RTCA/DO-178C.

[22] "Software considerations in airborne systems and equipment certification," 1992. Document No RTCA/DO-178B.

[23] S. Jacklin, "Closing the certification gaps in adaptive flight control software," in *Guidance, Navigation and Control Conference and Exhibit*, AIAA, 2008.

[24] A. Tasora, R. Serban, H. Mazhar, A. Pazouki, D. Melanz, J. Fleischmann, M. Taylor, H. Sugiyama, and D. Negrut, "Chrono: An open source multi-physics dynamics engine," in *High Performance Computing in Science and Engineering – Lecture Notes in Computer Science* (T. Kozubek, ed.), pp. 19–49, Springer, 2016.